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THE USE OF AN EDDY VISCOSITY MODEL TO PREDICT THE HEAT TRANSFER AND PRESSURE DROP PERFORMANCE OF ROUGHENED SURFACES

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Abstract—Calculations have been made of the velocity and temperature profiles for fully-developed turbulent flow in annular channels with roughened core rods. A unique set of boundary values for dimensionless velocity and temperature near the rough surface, together with a universal eddy viscosity model, serve to predict the friction factor and Stanton number of one particular surface in a wide variety of flow channels. Comparison with experimental data for nominally similar surfaces suggests that the method is accurate enough for practical purposes.

NOMENCLATURE

B, parameter in logarithmic velocity distribution;
Cp, specific heat of fluid at constant pressure;
d, diameter;
de, equivalent diameter;
δ, boundary layer thickness;
e, rib height;
e^+, non-dimensional rib height or rib Reynolds number, $\dfrac{eu_{\tau_1}}{\nu}$;
f, friction factor;
k_t, turbulent eddy conductivity;
K, universal constant in logarithmic inner law;
ν, kinematic viscosity of fluid;
ν_t, turbulent eddy viscosity;
P, function defined by equation (2.5.2);
p, rib pitch;
Pr, Prandtl number;
Pr_t, turbulent Prandtl number;
Re, Reynolds number;
r, radius;
r_0, radius of zero shear surface;
ρ, density of fluid;
St, Stanton number;
T, fluid temperature;
T^+, non-dimensional fluid temperature, $(T_w - T)\dfrac{\rho u_\tau Cp}{q''_w}$, where subscript w refers to wall values;
U^+ non-dimensional fluid velocity, $\dfrac{U}{u_\tau}$;
u_τ, friction velocity, $\sqrt{\dfrac{\tau_w}{\rho}}$, where τ_w is the wall shear stress;
y, distance from the effective origin of a surface.

Subscripts

$_{1/2}$, associated with inner/outer wall, either directly or by transformation.

1. INTRODUCTION

THE DEVELOPMENT of ribbed heat transfer surfaces for the fuel elements of the Advanced Gas-Cooled Reactor has demanded a method of comparing the thermal performance measured in single-pin tests. The measurements have been made with various heights of ribbing on various diameters of pin in various diameters of smooth channel. The required method must therefore relate the measured performance to a common geometry of flow channel.

The transformation due to Hall [1] defines a friction factor

$$f_1 \equiv \frac{\tau_{w_1}}{\frac{1}{2}\rho U_1^2},$$

where U_1 is the bulk velocity of the fluid in the inner region between the pin, of radius r_1, and the surface of zero shear stress, of radius r_0. In general, for a given geometry of roughening

$$f_1 \equiv f_1(e, r_1, r_0, U_1, \nu) \tag{1.1}$$

which may be written

$$f_1 \equiv f_1(e/de_1, d_1/de_1, Re_1), \tag{1.2}$$

where

$$de_1 \equiv \frac{2(r_0^2 - r_1^2)}{r_1}, \; d_1 = 2r_1$$

and e is the rib-height. Because of experimental uncertainties, particularly in the manufacture of geometrically similar ribbed surfaces, the dependence of f_1 on these parameters is difficult to define (see Lee [2]).

However on the hypothesis that the flow away from the immediate vicinity of the ribs is governed only by the local shear stress distribution in fully-developed flow, it is possible to predict this dependence. More precisely, for a particular shear stress distribution (defined in fully-developed flow by r_0, r_1 and r_2), the mean velocity profiles, when normalized by the appropriate friction velocity, u_τ, will be similar, whatever the character of the roughness generating the shear profile. This is an extension of the principle of Reynolds number similarity and is implied in the results of Liu *et al.* [3], who showed that for a boundary layer of thickness, δ, over a variety of rough surfaces, the distribution of $v_t/\delta u_\tau$, where v_t is the turbulent eddy viscosity, was a unique function of y/δ. A further extension of the principle is suggested by the data of Jonsson and Sparrow [4], who demonstrated that, to a good approximation, the eddy viscosity distribution in confined flows is a universal one, independent of the shear profile, if δ is taken to be the distance of the surface from that of zero shear.

It is noted in passing that this result is incompatible with the hypothesis of a universal velocity distribution, for if the radius ratio of the flow annulus, r_1/r_0, is small, the shear stress distribution has significant curvature, so that the velocity distribution differs from that in a pipe. Such a hypothesis forms the basis of a calculation method recently published by Maubach [5], but it is clearly belied by the velocity profiles measured by Lee [2].

In this work, calculations of friction factor and Stanton number are made using eddy viscosity distributions which are invariant with respect to surface roughness, together with "boundary values" of dimensionless velocity and temperature. For a given surface roughness, these boundary values are usually unaffected by the distribution of velocity and temperature in the bulk of the fluid: once they are determined by matching the computational results to those of a single experiment, the performance of that surface in any concentric flow annulus may be predicted.

2. CALCULATIONS

2.1 *Calculation procedure*

The calculations were performed on an IBM 360 using a program, code-name CONAN, for fully-developed incompressible turbulent flow in concentric annuli. The program is based upon one originally written by Ying [6] and later developed by Durst [7], and it has been shown by Lawn and Elliott [8] to agree well with hydrodynamic experimental data when applied to the smooth annulus.

In essence, the program selects a Reynolds number and estimates from an empirical correlation the friction factor associated with the outer wall. A first estimate of r_0 (in the presence of a rough inner wall) is that it lies $0{\cdot}3(r_2 - r_1)$ from the outer wall, from which the wall shear stresses, and hence eddy viscosity distributions (see 2.2), for the inner and outer regions can be derived. Integration proceeds from both walls so that, using the appropriate boundary values (see 2.5), velocity distributions up to the zero shear surface are obtained, and the amount by which they fail to match is used as a criterion for the adjustment of r_0 for the next calculation. Iteration continues until the velocities match and then the exact value of the Reynolds number for that velocity distribution is calculated. If this does not correspond to the Reynolds number specified, the outer wall friction factor is adjusted and the calculation is repeated until the correspondence is as close as desired.

Heat transfer with uniform flux from the core-rod may then be described by calculation of the temperature from an eddy conductivity distribution (see 2.3), and integration of the energy equation to obtain the heat flux distribution. The variation of physical properties is neglected, so the results must be compared with data extrapolated to a wall-to-bulk temperature ratio of unity.

2.2 *Eddy viscosity model*

Jonsson and Sparrow [4] have shown that a correlation of the form:

$$\frac{v_t}{|r_0 - r_{1,2}|u_{\tau_{1,2}}} = F\left(\frac{y_{1,2}}{|r_0 - r_{1,2}|}\right) \qquad (2.2.1)$$

provides an accurate representation of the variation of eddy viscosity in each of the two flow regions of an all-smooth annulus. A ramp function, found to be successful for moderate radius ratios by Lawn and Elliott [8], was also used here for the annulus with a rough core-rod. Thus:

$$\frac{v_t}{(r_0 - r_1)u_{\tau_1}} = 0{\cdot}325\left(\frac{y_1}{r_0 - r_1}\right) \quad \text{for } \frac{y_1}{r_0 - r_1} < 0{\cdot}233$$
$$= 0{\cdot}325 \times 0{\cdot}233 = 0{\cdot}076 \quad \text{for } 0{\cdot}233 < \frac{y_1}{r_0 - r_1} < 1, \qquad (2.2.2)$$

and similarly for the outer flow region.

The discontinuity of v_t at r_0, present in the model, was eliminated by Durst [7], by allowing the effective viscosity on each side of r_0 to be influenced by the other.

In the present work, it was found that the best fit to experimental data was obtained by allowing the smooth surface viscosities to be influenced by the rough but not vice-versa. In view of the much greater flow area associated with the rough wall, this seemed a reasonable modification.

2.3 *Eddy conductivity*

The eddy conductivity, k_t, was calculated from the eddy viscosity, assuming a constant value for turbulent Prandtl number,

$$Pr_t = \frac{\nu_t}{k_t}. \qquad (2.3.1)$$

On the evidence of Gowen and Smith [9], $Pr_t = 0{\cdot}90$ was chosen for the present calculations in which $Pr = 0{\cdot}745$.

2.4 *The origin of co-ordinates for rough surfaces*

The distances y_1 and y_2 are here the effective distances from the surfaces, obvious in the case of the smooth wall, but ambiguous in the case of the rough wall. The origin of y_1 is often taken to be either the crest or the root of the ribs. Neither of these definitions is necessarily consistent with a logarithmic velocity profile of universal gradient in regions of constant stress, such as would be predicted by (2.2.2). In fact, the constant of 0·325 in (2.2.2) is very much smaller than the value of $K \simeq 0{\cdot}40$ normally used in pipe flow. There are two reasons for this. In the first place, the expression is the "best fit" ramp function for ν_t: a greater gradient close to the wall and a smaller one as $y/(r_0 - r_1) \to 0{\cdot}233$ would probably be an even better fit, but this would destroy the simplicity of the expression. Secondly, the results of Lawn and Elliott [8] for smooth annuli suggest that a lower value of gradient than that for a pipe is appropriate for annular passages of moderate radius ratio.

It was found by Lawn and Hamlin [10] that a universal logarithmic inner law is not observed for surfaces with transverse square ribs of pitch-to-height ratio 7·2, if the root of the ribs is taken as the origin, although the analysis in that work admittedly included velocity measurements outside the "constant stress" layer. Further analysis of that data and tests on really large-scale ribbed surfaces have indicated that a displacement of the origin by one rib height, e, below the root is necessary to fit the data to the universal logarithmic inner law:

$$U^+ = \frac{1}{K}\ln\frac{y_1}{e} + B. \qquad (2.4.1)$$

This displacement of origin for $p/e = 7{\cdot}2$ may be compared with the results of Perry and Joubert [11], Liu *et al.* [3], and Bettermann [12], all of whom found the effective origin to lie between the crest and root of the rib for $p/e = 4$, and those of Hanjalic and Launder [13], who found a displacement of $0{\cdot}4e$ below the root for $p/e = 10$. That a surface with $p/e \sim 7$ should have the deepest origin might be expected, for although it has a large enough inter-rib distance for flow reattachment (it is not a cavity flow), this distance is not large enough for the individual ribs to be effectively isolated. According to Abbott and Kline [14], the reattachment point lies $8e$ behind an isolated rib. It is this property that gives maximum frictional resistance to surfaces with $p/e \sim 7$, as shown by Wilkie [15].

2.5 *Boundary values*

The eddy viscosity distributions (2.2.2) were discontinued before the walls were reached. Near the smooth outer wall, this occurred at $y_2 u_{\tau_2}/\nu = 30$ and from there to the surface, the van Driest [16] eddy viscosity distribution was used. This effectively gives the correct value of U_2^+ at $y_2^+ = 30$ and so may be regarded as being equivalent to a boundary value, although of course the true boundary condition is $U_2^+ = 0$ at $y_2^+ = 0$.

Near the rough inner wall, the situation is complicated by the variations in velocity in the direction of flow as it goes over the ribs. Again from the results of work on large-scale ribs, it has been found that these variations become negligible $3e$ from the rib tips ($5e$ from the effective origin) and so a value of U^+ at $y_1/e = 5$, which will apply uniformly along the boundary, may be specified. This is of course equivalent to specifying the value of B in (2.4.1). For surfaces with transverse square ribs and $p/e = 7{\cdot}2$,

$$U^+_{(y_1 = 5e)} = 6{\cdot}80 \qquad (2.5.1)$$

was chosen, implying $B = 2{\cdot}96$ if $K = 0{\cdot}419$, as suggested by Patel [17]. This should hold for sufficiently high e^+ and sufficiently low e/de_1.

The resistance to heat transfer of the flow between the ribs and $y_1/e = 5$ was expressed in the form of a value of $T^+ \equiv (T_w - T)\rho u_{\tau_1} Cp/\dot{q}''_w$ at $y_1/e = 5$. This value was calculated from a "P function", of the type proposed by Jayatillaka [11], which is defined by:

$$T^+ = Pr_t(U^+ + P), \qquad (2.5.2)$$

and here $U^+ = 6{\cdot}80$.

Analysis of data for three-dimensional roughness elements led Jayatillaka to a correlation:

$$P \propto Pr^{0{\cdot}70} e^{+0{\cdot}36}. \qquad (2.5.3)$$

It was assumed that a correlation of the same form would apply to two-dimensional elements. The best fit to experimental data for $Pr = 0{\cdot}745$ (CO_2) was obtained with:

$$P = 1{\cdot}72\, Pr^{0{\cdot}70} e^{+0{\cdot}36}. \qquad (2.5.4)$$

Although no data is available to test the Prandtl number dependence for this case, the dependence on e^+ was confirmed for one particular value of p/e (see section 3.2).

It was also necessary to specify the convective heat and mass fluxes through the flow region next to the ribbed surface, although these are not critical parameters if the ribs are small in comparison with the total flow area.

3. COMPARISON WITH EXPONENTAL DATA

3.1 *Isothermal data*

The data chosen for comparison with the calculated values of friction factor are those of Lee [2] obtained in an air rig. Lee initially tested six pins of differing diameters with transverse-ribbed roughness elements, all of the same type but of different height, in four different diameters of smooth channel. The method used to obtain the transformed values of friction factor and equivalent diameter, quoted in Table 1, is described in Lee's paper. Attention is here concentrated upon the untransformed friction factors, however; these were taken from the original data. The values at $Re_1 = 6 \times 10^5$ and $Re_2 = 10^5$ were each interpolated from about six results obtained over a range of Reynolds numbers.

It is seen from Table 1 that, after the various adjustments specified in section 2, CONAN consistently predicts both the overall friction factor and the surface of zero shear stress, or de_1, (with standard deviations of about 3 per cent and 2 per cent respectively) for the very wide range of flow geometries tested by Lee. One surface, No. 3, was in fact excluded from the averaging, because in one test the results showed more scatter than usual, and in both tests, the discrepancy between measured and predicted values of f_1 was much larger than the standard deviation, suggesting that the surface was atypical, probably in having slightly more rounded ribs than the others.

The origin of this discrepancy is found in the rough surface velocity profiles, Figs. 1 and 2. Whereas the boundary value $U_{(\bar{y}_1 = 5e)} = 6{\cdot}80$, is a good fit to the results for surfaces 1, 4, 5 and 6, surface 3 requires a rather higher value, and so too (but to a lesser extent) does surface 2. This is manifested in predicted values of f_1 which are too large (Table 1).

Also apparent from Figs. 1 and 2 is that it is only when the zero shear surface is about $40e$ distant from the rib that any substantial portion of the profile varies as (2.4.1) with $K = 0{\cdot}419$. This contradicts the assumption of Maubach [5] regarding the universality of the profiles. If $(r_0 - r_1) \sim 20e$, then the approximately constant stress layer barely exists outside the region of axial velocity variations generated by the ribs.

For each of the surfaces, the scatter in the measured values of f_2 is large, so that the interpolated values for $Re_2 = 10^5$ are uncertain to ± 2 per cent. The average discrepancy between them and the predicted values of $-0{\cdot}3$ per cent therefore demonstrates that the complicated effect of geometry upon the smooth surface velocity profile beyond the universal inner law region, is adequately handled by the eddy diffusivity model, which includes an enhancement due to the influence of the high values of diffusivity associated with the rough wall opposite.

Friction factors for four of the test results, embracing the extreme cases of large and small ribs in large and small radius ratio channels, are plotted against Reynolds number in Fig. 3. The predicted Reynolds number variation over the limited range of the tests is seen to be compatible with the experimental results in all four cases.

Table 1. Friction factor predictions: data of Lee [2]

					$Re_1 = 6 \times 10^5$						$Re_2 = 10^5$	
					f		e/de_1		f_1		f_2	
Test No.	Surface No.	Rib height e (mm)	e/r_1	r_1/r_2	Measured	Predicted	Measured	Predicted	Measured	Predicted	Measured	Predicted
1	1	1·42	0·0378	0·5209	0·0127	0·0130	0·00958	0·00981	0·0281	0·0292	0·0051	0·0050
2	2	1·42	0·0378	0·5209	0·0127	0·0130	0·00958	0·00981	0·0281	0·0292	0·0051	0·0050
3*	3	0·68	0·0257	0·3695	0·0085	0·0084	0·00340	0·00341	0·0180	0·0194	0·0052	0·0049
16	4	0·38	0·0220	0·2375	0·0063	0·0063	0·00143	0·00142	0·0147	0·0152	0·0046	0·0047
5	2	1·42	0·0378	0·4300	0·0102	0·0108	0·00639	0·00643	0·0233	0·0253	0·0049	0·0049
6	1	1·42	0·0378	0·4300	0·0106	0·0108	0·00635	0·00643	0·0238	0·0253	0·0049	0·0049
7*	3	0·68	0·0257	0·3051	0·0073	0·0075	0·00253	0·00243	0·0162	0·0177	0·0050	0·0048
12	4	0·38	0·0222	0·1961	0·0060	0·0058	0·00106	0·00107	0·0155	0·0144	0·0046	0·0047
18	4	0·38	0·0222	0·3292	0·0077	0·0075	0·00246	0·00243	0·0178	0·0172	0·0047	0·0048
11	5	1·42	0·0830	0·1961	0·0081	0·0080	0·00320	0·00327	0·0257	0·0252	0·0047	0·0049
13	6	0·38	0·0083	0·5230	0·0078	0·0078	0·00260	0·00251	0·0152	0·0148	0·0050	0·0049
14	6	0·38	0·0083	0·6334	0·0100	0·0094	0·00403	0·00405	0·0177	0·0177	0·0051	0·0049
15	5	1·42	0·0830	0·2375	0·0089	0·0089	0·00438	0·00441	0·0267	0·0268	0·0049	0·0049
19	5	1·42	0·0830	0·3292	0·0111	0·0112	0·00785	0·00784	0·0293	0·0307	0·0052	0·0050
		Average % Discrepancy			0·0		+0·5		+1·2		−0·3	
		% Standard Deviation			3·1		1·6		4·8		2·6	

* Excluded from averaging.

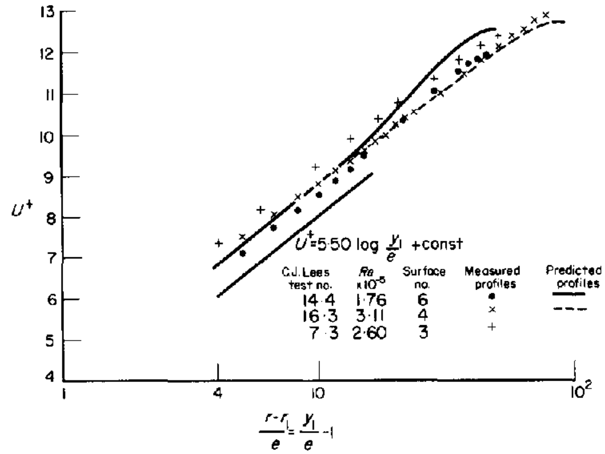

FIG. 1. Comparison of measured and predicted rough surface velocity profiles.

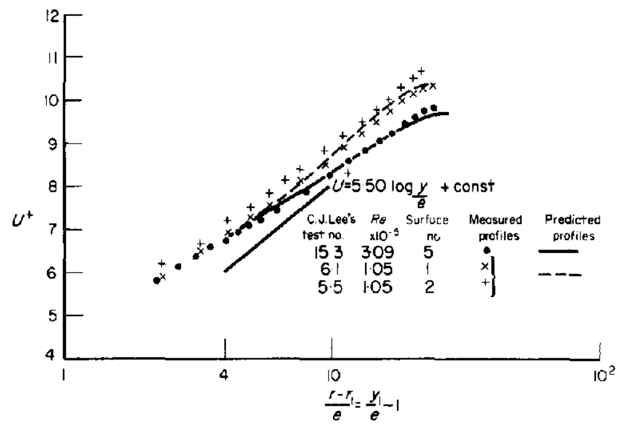

FIG. 2. Comparison of measured and predicted rough surface velocity profiles.

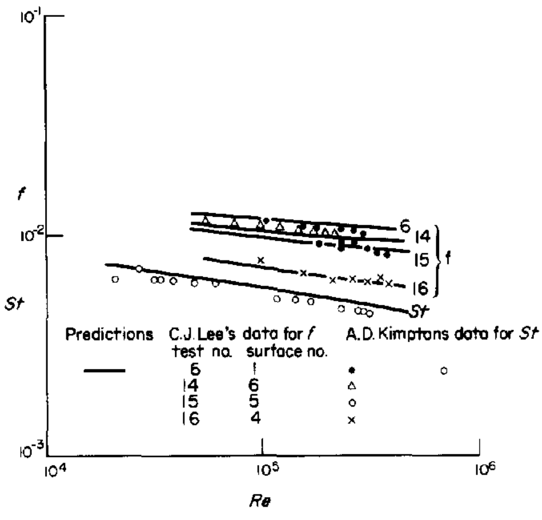

FIG. 3. Reynolds number variation of friction factor and Stanton number.

3.2 *Heated pin data*

The data are those of Kimpton and Lyall [19], Wilkie [15], Watson [20] and White and White [21]. The basis of comparison is a situation of infinite thermal conductivity in the ribbed surface, a wall-to-bulk temperature ratio of unity, a Prandtl number of 0·745 appropriate to CO_2 and sharp rib profiles. Details of the corrections for finite thermal conductivity and other Prandtl numbers are given by Mantle *et al.* [22], for wall-to-bulk temperature ratios by Kimpton and Lyall [19], and for rib rounding by White and White [21].

Transformed results only are presented by Wilkie [15] and White and White [21], but another paper by Wilkie [23] gives details of the transformation and allows the overall Stanton numbers to be recovered. Kimpton's untransformed results were taken from the

Table 2. Stanton number predictions: data correlated by Kimpton and Lyall [19]

Reference	Rib height e (mm)	e/r_1	r_1/r_2	$Re_1 = 8 \times 10^5$ St Measured	St Predicted
Kimpton and Lyall [19]	0·26	0·0348	0·4300	0·00430	0·00469
Wilkie [15]	0·41	0·0172	0·4606	0·00397	0·00435
	0·63	0·0260		0·00433	0·00456
	0·78	0·0322		0·00451	0·00465
	1·02	0·0422		0·00494	0·00471
	1·25	0·0518		0·00531	0·00475
Watson [20]	0·46	0·0172	0·5122	0·00438	0·00438
	0·68	0·0256		0·00462	0·00456
	1·14	0·0428		0·00500	0·00465
White and White [21]	0·24	0·0234	0·4629	0·00433	0·00452
	0·28	0·0270		0·00460	0·00458
	0·37	0·0358		0·00456	0·00468
	0·37	0·0364		0·00470	0·00468
	0·48	0·0466		0·00490	0·00472
	0·56	0·0546		0·00486	0·00475
	0·60	0·0586		0·00483	0·00476
	0·65	0·0636		0·00510	0·00480

Average % Discrepancy −0·2
% Standard Deviation 5·5

original data. It is these untransformed values that are compared in Table 2 with St calculated by CONAN for $Re_1 = 8 \times 10^5$, since the program does not perform the Stanton number transformation.

Once more the agreement is satisfactory in comparison with the probable experimental errors and the uncertainty in the corrections to the data. As Kimpton's own untransformed Stanton number results are available for a wide range of Reynolds numbers (he tested the same pin in both air and CO_2), they are plotted in Fig. 3 to show that the predicted Stanton number variation is also correct. This implies that the dependence of the boundary values on e^+ is correct, provided e^+ is greater than 30, and perhaps for even smaller values, although B is likely to vary at low Reynolds number if the ribs are rounded.

The difficulty of quantifying the extent to which ribs were rounded and the effect of the rounding upon the measured Stanton numbers in fact introduces considerable uncertainty into the comparison with the predictions. Another factor is that some of the surfaces only nominally had $p/e = 7{\cdot}2$: Watson's [20] largest ribs were in fact pitched at $p/e = 6{\cdot}5$ and Kimpton's at $p/e = 7{\cdot}5$, which may account for the particularly large discrepancies in those cases. In addition, it should be remarked that in several of the experiments the flow passage was less than 20 rib heights wide, so the boundary values may have been influenced by the outer flow.

4. CONCLUSION

Implicit in the procedure of section 2·1 is the generation of a velocity profile with a maximum at the zero shear surface. This is known to be incorrect but because the velocity distribution in the core has only a secondary effect on the integral flow-parameters, it is an admissible defect in this work.

The crucial parameters in the calculation method are of course the boundary values for velocity and temperature. However, as the effects of duct geometry are satisfactorily accounted for by the method, detailed velocity measurements are not required to define the functions B and P. For any particular surface, one test over a range of Reynolds numbers should be sufficient to establish by trial and error from the overall friction factor data, the appropriate functional dependence of B on e^+, and this relation can be used in all subsequent calculations. Similarly, Stanton number data can be compared with predictions to define $P(e^+)$ for the particular surface.

If the ribs are sharp and transverse to the flow and the rib Reynolds number is greater than 30, a value of B independent of e^+, and $P \propto e^{+0{\cdot}36}$, may be assumed for the surface. For one surface of this type, on which the ribs are square and pitched 7·2 rib heights apart, the calculation method has been shown to adequately predict the velocity and temperature profiles in the annular flow passage around various diameters of core-rod roughened by various heights of rib.

Integral parameters were predicted with a standard deviation of about 5 per cent. Greater error may be expected if the flow passage is less than about 20 rib heights wide but the major part of the error in the present comparison is probably due to lack of geometrical similarity in the actual surfaces.

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UTILISATION D'UN MODELE DE VISCOSITE TURBULENTE POUR L'ESTIMATION DU TRANSFERT THERMIQUE ET DE LA PERTE DE CHARGE POUR DES SURFACES RUGUEUSES

Résumé—Des calculs de profils de vitesse et de température ont été faits pour un écoulement turbulent entièrement développé dans des canaux annulaires avec des barres centrales rugueuses. Un ensemble unique de valeurs limites pour la vitesse et la température sans dimension près de la surface rugueuse, liées toutes deux par un modèle universel de viscosité turbulente, est utilisé pour l'estimation du coefficient de frottement et du nombre de Stanton d'une surface particulière dans une large variété de canaux d'écoulement. Une comparaison avec des résultats expérimentaux pour des surfaces à peu près similaires montre que la méthode est suffisament précise pour des buts pratiques.

DER GEBRAUCH EINES SCHEINREIBUNGSMODELLS ZUR BESTIMMUNG VON WÄRMETRANSPORT UND DRUCKABFALL AN RAUHEN OBERFLÄCHEN

Zusammenfassung—Berechnungen der Geschwindigkeit und der Temperaturprofile für eine voll ausgebildete turbulente Strömung in ringförmigen Kanälen mit Stabbündeln wurden durchgeführt. Ein einziger Satz von Grenzwerten für die dimensionslose Geschwindigkeit und die Temperatur in der Nähe der rauhen Oberflächen dient, zusammen mit einem universalen Scheinreibungsmodell, der Vorhersage des Reibungsfaktors und der Stanton-Zahl für eine bestimmte Oberfläche bei einer großen Zahl von Strömungskanälen. Vergleiche mit experimentellen Daten für nominell ähnliche Oberflächen zeigen, daß die Methode für praktische Zwecke genügend genau ist.

ИСПОЛЬЗОВАНИЕ МОДЕЛИ ТУРБУЛЕНТНОЙ ВЯЗКОСТИ ДЛЯ РАСЧЁТА ТЕПЛООБМЕНА И ПЕРЕПАДА ДАВЛЕНИЯ ПРИ ТЕЧЕНИИ МЕЖДУ ШЕРОХОВАТЫМИ ПОВЕРХНОСТЯМИ

Аннотация — Проведен расчёт профилей скорости и температуры для полностью развитого турбулентного течения в кольцевых каналах с шероховатыми стержнями. Универсальная система пограничных условий для безразмерной скорости и температуры у шероховатой поверхности, а также универсальная модель турбулентной вязкости служат для расчёта коэффициента трения и числа Стантона одной отдельной поверхности множества каналов. Сравнение с экспериментальными данными для подобных поверхностей свидетельствует о том, что описанный метод достаточно точен и пригоден для практического использования.